Hats and pancakes in the sky: high-speed droplet dynamics

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(I) Motivation and aims

What are droplet dynamics?	→	Study of fluid droplets evolving in a secondary fluid phase (e.g. water droplets in air).
Applications	→	3D/ink-jet printing, spray technologies, forensics, virus spreading (coughing/sneezing) and aeronautics etc.
Motivation	→	Accurately model water droplet dynamics prior to/after impact with incoming aerofoil → danger of aircraft icing.
Challenges	→	Two-phase fluid, small droplets, large aerofoil, high-speed airflow, large density/viscosity ratios → difficult to model analytically, experimentally and numerically.

What did we do?

Analysed an existing droplet trajectory and deformation model
(simplistic but currently the most advanced analytical model).

Developed a high resolution predictive droplet model
→ via direct numerical simulations (DNS).

Used DNS results to assess validity and accuracy of assumptions in
the existing model.

Take home message

Highly challenging to capture/predict pre-impact dynamics in this flow regime –> goal is to develop predictive models (using DNS) that capture trajectory, non-spheroidal deformations and breakup.

(II) An existing droplet model

Origin

- Developed by Sor et al.[1] → experimentally informed force-balance model → no fluid mechanics.

$$m\frac{d^2x}{dt^2} = -F_{D_x}$$
$$m\frac{d^2y}{dt^2} = F_{D_y} - mg$$
$$m\frac{d^2a}{dt^2} = F_p - F_{st} - F_v$$

Overview

- Two-dimensional flow → aerofoil moving at constant speed (flow in front of droplet accelerates) .
- Trajectory (x,y) and deformation (a) tracked → via force balance equations.
- Taylor analogy → deformation modelled like a mass-spring system (harmonic oscillator).

Assumptions

1. Vertical air flow negligible.
2. Droplet deforms as oblate spheroid (see Fig.1).
3. No breakup of droplet.

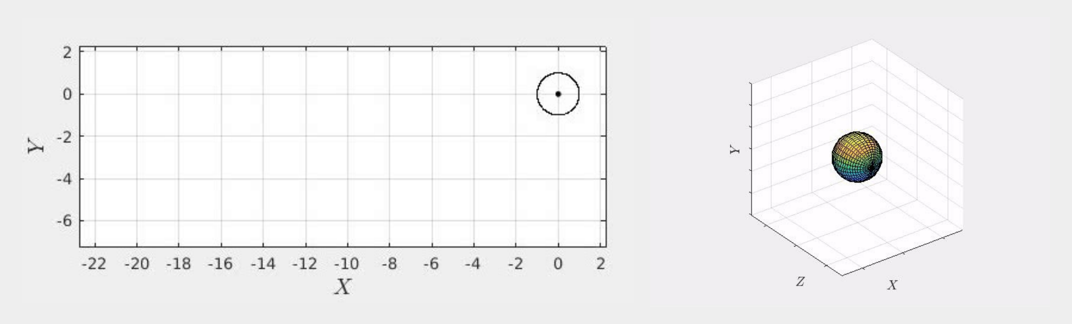


Fig. 1: (Left) Trajectory and deformation of droplet. (Right) 3D rotating view tracking the droplet.

[1] S. Sor, A. García-Magariño, and A. Velazquez, "Model to predict water droplet trajectories in the flow past an airfoil," Aerospace Science and Technology 58, 26–35 (2016).

(II) An existing droplet model

What did further analysis identify?

- X trajectory varies drastically for small changes in initial velocity.
- Y trajectory can be ignored → gravity has little effect and droplets typically suspended.
- Pressure battles surface tension force → driving the oscillations and deformation.
- Larger droplets oscillate less but deform most - uncharacteristic as breakup would occur.

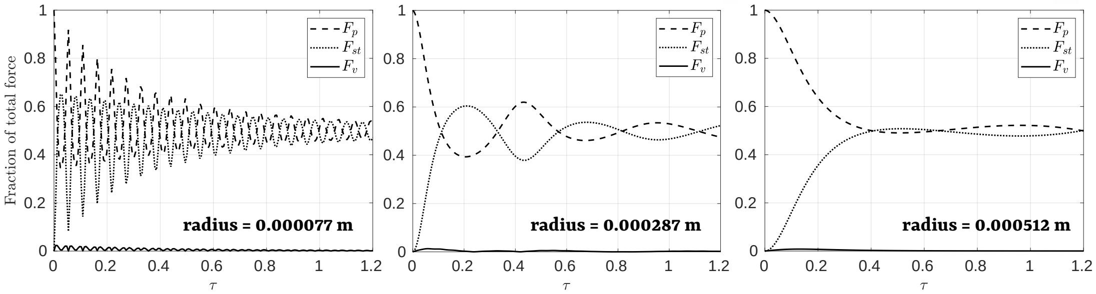


Fig. 4: Relative force contributions to deformation vs. time for various droplet radii.

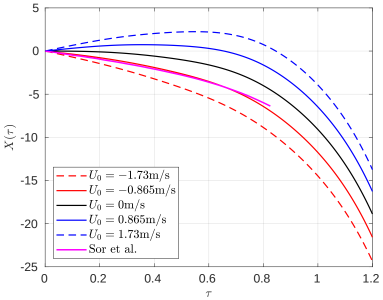


Fig. 2: Effect of initial horizontal velocity on X trajectory vs. time.

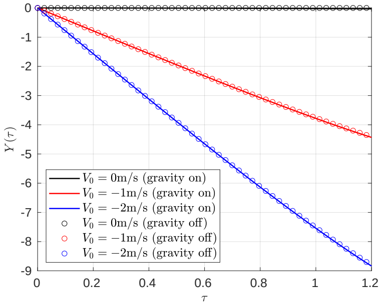


Fig 3: Effect of initial vertical velocity (and gravity) on Y trajectory vs. time.

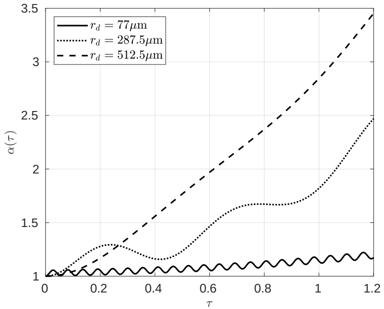


Fig. 5: Vertical deformation vs. time for varying droplet radii.

(III) Numerical model (DNS)

Overview

- Axisymmetric half-droplet → governed by two-phase Navier-Stokes equations + interface conditions → open-source C library *Basilisk.*

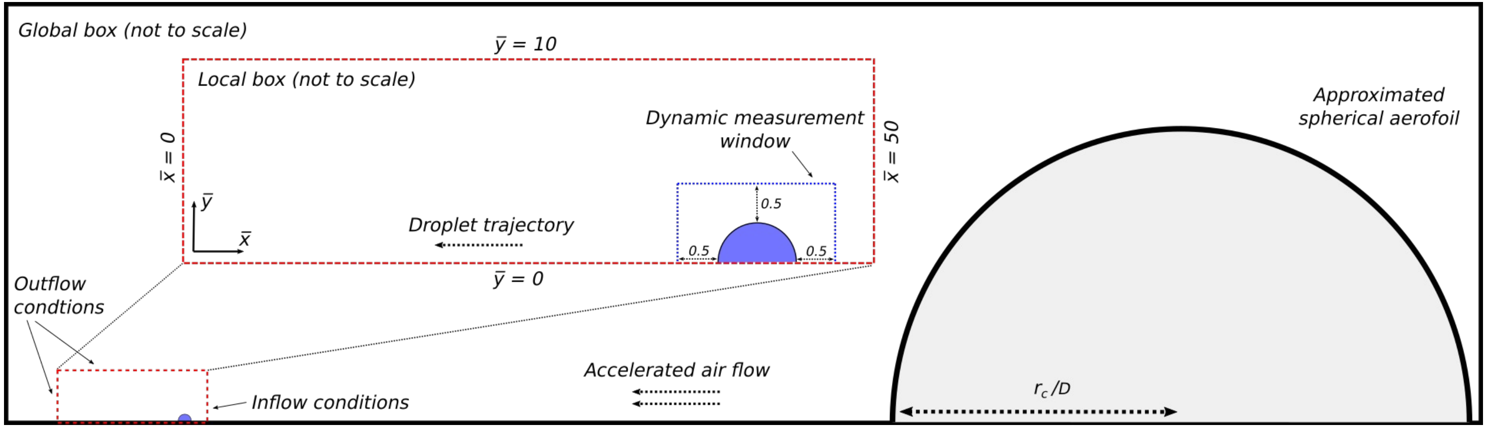


Fig. 6: Global and local computational boxes.

- Global model → too computationally heavy → requires > $O(10^7)$ grid points.
- Local model → still multi-scale, requiring tens of CPU hours solving in parallel but tractable → requires outflow and inflow conditions.
- Inflow conditions found solving global model *without* droplet → computationally inexpensive (alternatively use analytical potential flow around sphere).

(III) Numerical model (DNS)

Trajectory and deformation results

- Droplet accelerates → initially oblate spheroidal shape → assumption breaks down at later times.
- No oscillations observed → calls Taylor (mass-spring) analogy into question in this particular case.

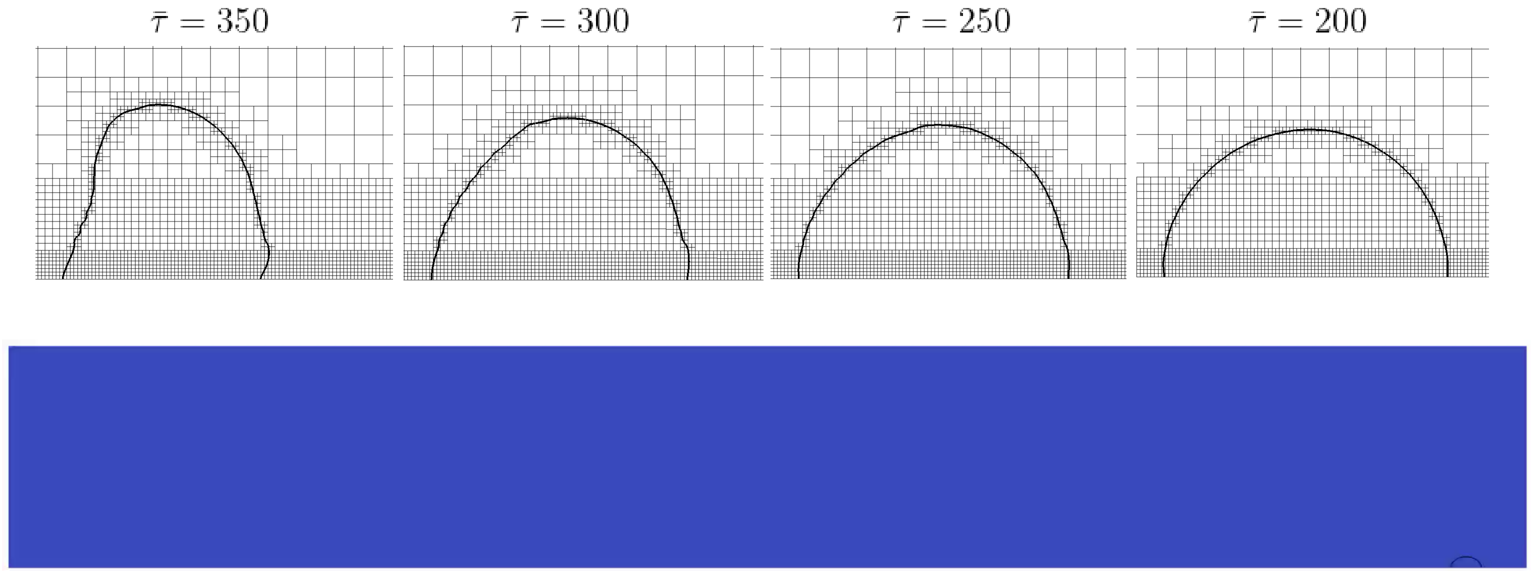


Fig. 7: Trajectory and deformation observed in the DNS model with horizontal velocity field (red = slow flow, blue = fast flow).

What did the flow analysis reveal?

- Flow measured in front, behind and above droplet.
- Non-spheroidal deformation driven by increasing pressure gradient across droplet.
- Negligible vertical velocity assumption verified above droplet center of mass.

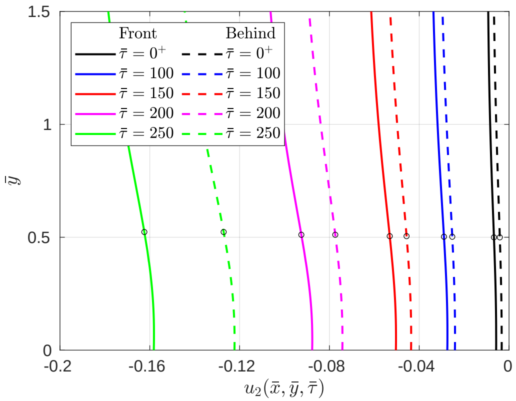


Fig. 8: Horizontal air velocity profile in front/behind droplet.

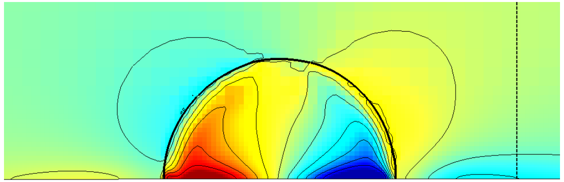
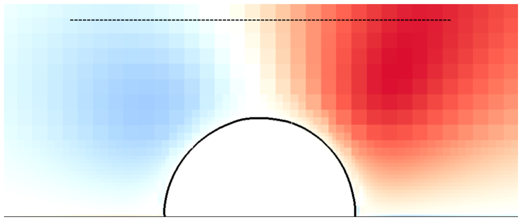


Fig. 9: Pressure field around/inside the droplet prior to deformation.



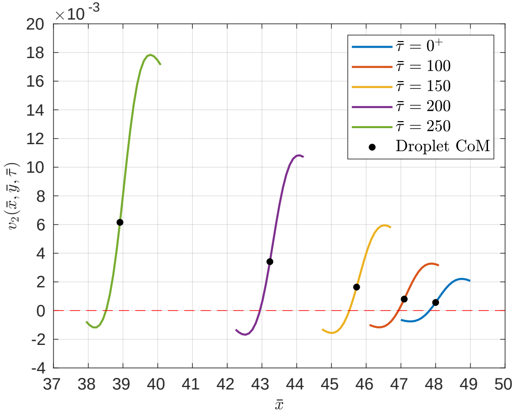


Fig. 10: (Top) Vertical velocity field around droplet prior to deformation. (Bottom) Velocity profiles above droplet.

(IV) Conclusions and future work

Existing model – what did we find?

- Vertical trajectory can be ignored → droplets typically suspended in clouds.
- Oblate spheroidal shape → holds up to certain time → cannot capture non-uniform deformation thereafter.
- Taylor analogy – no shape oscillations found in this regime → needs further investigation.
- Verified negligible vertical background flow in stagnation region.
- Difficult to re-create results → heavy reliance on experimental parameters → hinders predictive power.

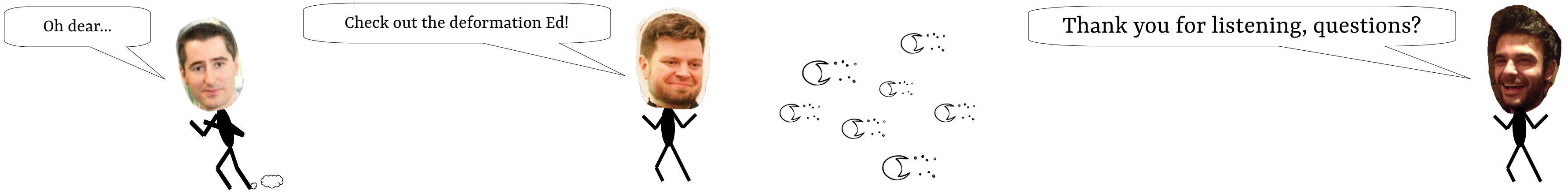
Numerical model – what did we achieve?

- Good first attempt at predicting pre-impact dynamics in violent flow regime → self contained predictive model.
- High-resolution flow → detailed deformation and flow quantities close to/within droplet.
- Efficient coupling of global and local domains → solvable on realistic timescales ($\mathcal{O}(10^2)$ CPU hours).

(IV) Conclusions and future work

Future work

- Further numerical validation of DNS over range of droplet sizes/flow conditions.
- Relax negligible vertical airflow assumption → considers droplets away from stagnation region.
- Investigate droplet breakup in accelerating flow vs. constant background flow.



Extra Slides

(II) An existing droplet model

Experimental setup

Fig: Experimental rotating arm facility used to verify the analytical model (taken from [1]).

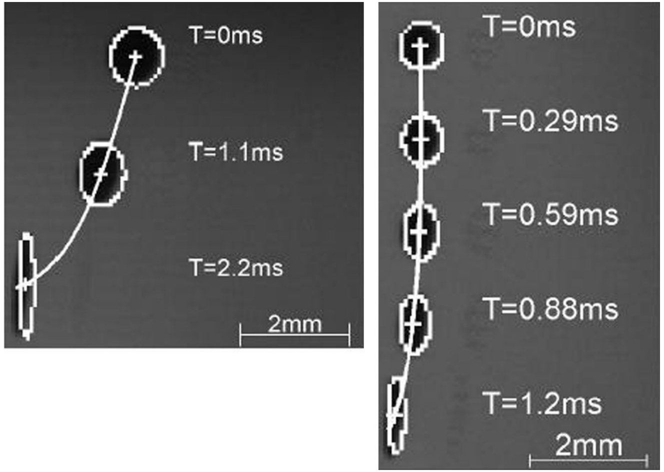


Fig: Analytical model results overlaid on images from the experiments [1].

[1] S. Sor, A. García-Magariño, and A. Velazquez, "Model to predict water droplet trajectories in the flow past an airfoil," Aerospace Science and Technology 58, 26–35 (2016).

(II) An existing droplet model

Governing equations

- Newton's second law → approximated forces incorporate: accelerating flow, drag laws, surface area change etc.
- Solve numerically using RK4 method.

Horizontal trajectory

$$m\frac{d^2x}{dt^2} = -F_{D_x}$$

Vertical trajectory

$$m\frac{d^2y}{dt^2} = F_{D_y} - mg$$

Perpendicular deformation

$$m\frac{d^2a}{dt^2} = F_p - F_{st} - F_v$$

Aerodynamic drag forces

Gravitational acceleration

Droplet mass

Pressure, surface tension and viscous forces

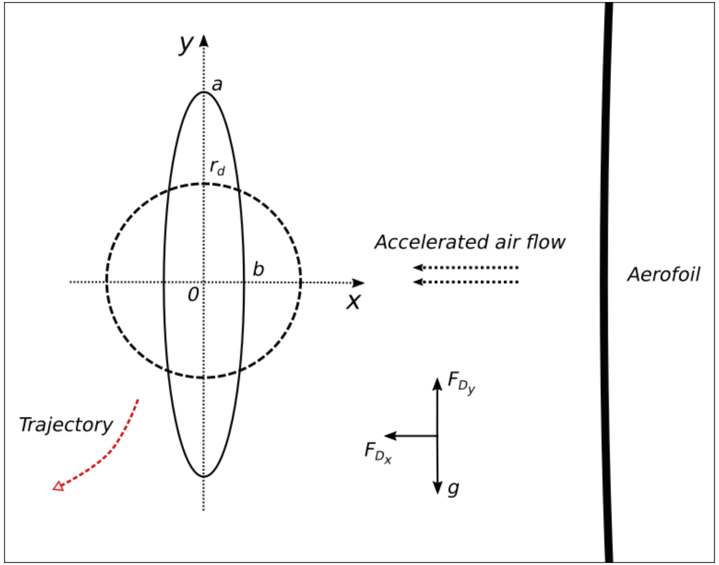
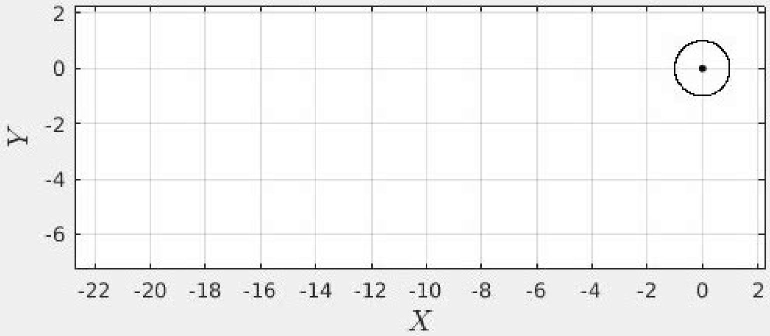


Fig: Cross-section of oblate spheroidal droplet with incoming aerofoil.

(II) An existing droplet model

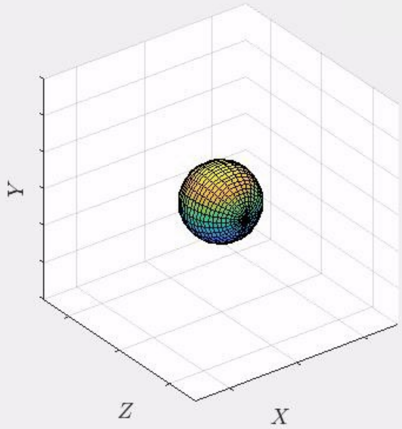
Trajectory and deformation

Fig: 2D cross-sectional view.



Droplet radius : 0.000287 m
Aerofoil velocity : 91 m/s
Initial droplet velocity (x,y): (-0.865, -1) m/s
Simulation time: 0.00136 s

Fig: 3D view tracking deformation.



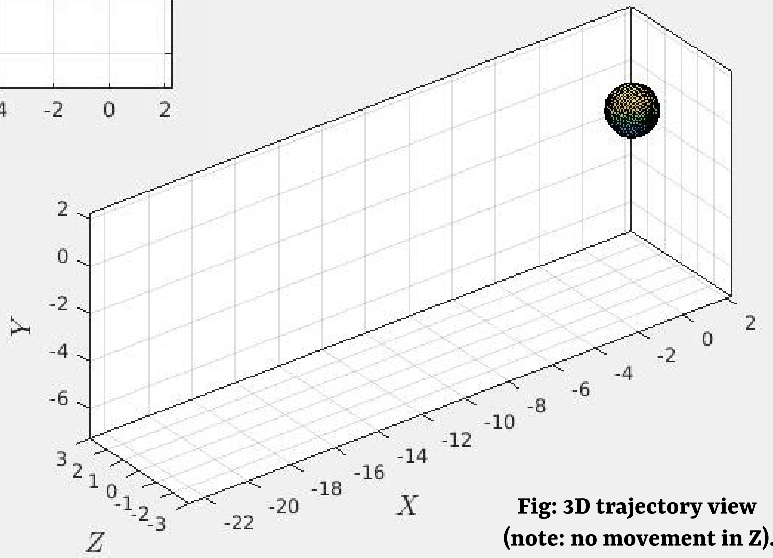


Fig: 3D trajectory view (note: no movement in Z).

(II) An existing droplet model

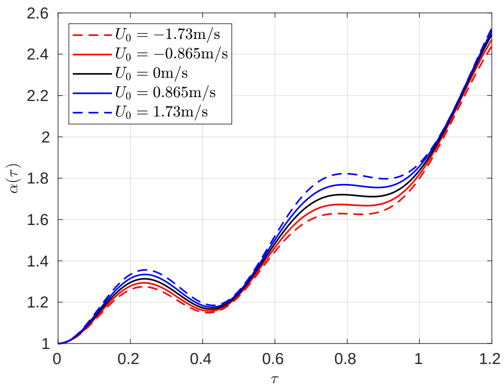


Fig: Deformation vs. time for droplets with varying initial horizontal velocity.

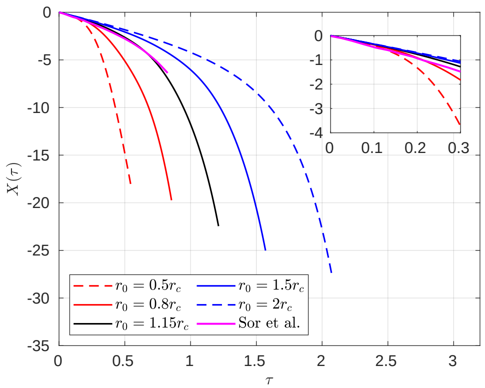


Fig: X displacement vs. time for droplets with varying initial distance from aerofoil.

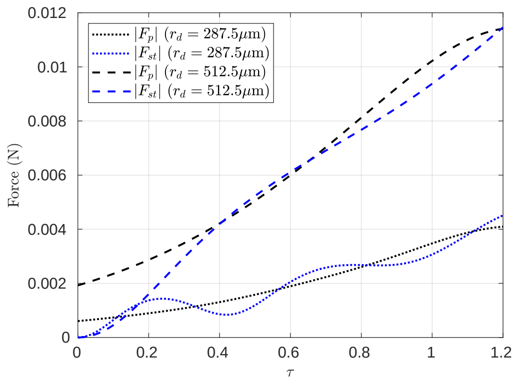


Fig: Force vs. time for droplets of varying radii.

(III) Numerical model (DNS)

Governing equations

- Two-phase fluid (water/air) + droplet interface between them.
- Requires robust numerical integrator → open-source library *Basilisk*[2] → second-order accurate solutions in space/time on adaptive meshes.

Dimensionless Navier-Stokes equations (in each fluid)

$$\boldsymbol{u}_{1\bar{\tau}} + (\boldsymbol{u}_1 \cdot \boldsymbol{\nabla})\boldsymbol{u}_1 = -\boldsymbol{\nabla} p_1 + \frac{1}{\mathrm{Re}_1}\boldsymbol{\nabla}^2 \boldsymbol{u}_1 - \frac{1}{\mathrm{Fr}^2}\boldsymbol{F}_g$$

$$\nabla \cdot u_1 = 0$$

$$\rho\Big(\boldsymbol{u}_{2\bar{\tau}} + (\boldsymbol{u}_2 \cdot \boldsymbol{\nabla})\boldsymbol{u}_2\Big) = -\boldsymbol{\nabla} p_2 + \frac{\mu}{\mathrm{Re}_1}\boldsymbol{\nabla}^2 \boldsymbol{u}_2 - \frac{\rho}{\mathrm{Fr}^2}\boldsymbol{F}_g$$

$$\nabla \cdot u_2 = 0$$

Interface equations on $\bar{y} = h(\bar{x}, \bar{\tau})$

$$u_1 = u_2$$

$$v_1 = h_{\bar{\tau}} + u_1 h_{\bar{x}}, \quad v_2 = h_{\bar{\tau}} + u_2 h_{\bar{x}}$$

$$\left[4\frac{\mu_i}{\mu} h_{\bar{x}} u_{i\bar{x}} + \frac{\mu_i}{\mu}\left(h_{\bar{x}}^2 - 1\right)\left(u_{i\bar{y}} + v_{i\bar{x}}\right)\right]_2^1 = 0$$

$$\left[-p_i(1+h_{\bar{x}}^2)+\frac{2}{\mathrm{Re}_1}\frac{\mu_i}{\mu}\left(h_{\bar{x}}^2 u_{i\bar{x}}+v_{i\bar{y}}-h_{\bar{x}}\left(u_{i\bar{y}}+v_{i\bar{x}}\right)\right)\right]_2^1 = \frac{1}{\mathrm{We}}\frac{h_{\bar{x}\bar{x}}^2}{\sqrt{1+h_{\bar{x}}^2}}$$

[2] S. Popinet, "An accurate adaptive solver for surface-tension-driven interfacial flows," Journal of Computational Physics 228, 5838–5866 (2009).

(III) Numerical model (DNS)

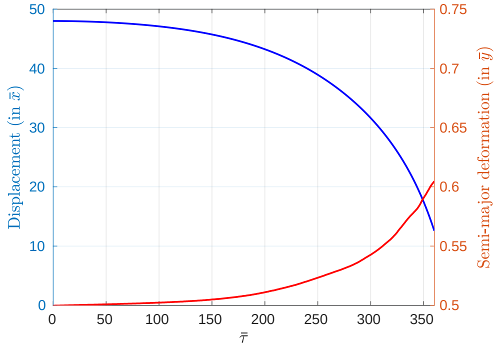


Fig: Displacement and deformation vs. time for droplet from DNS model.

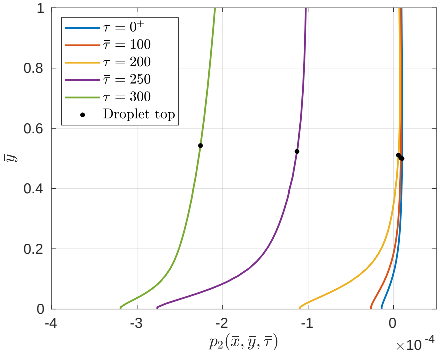


Fig: Air pressure profiles in front of droplet at increasing times.