Validation of the static forward Grad-Shafranov equilibrium solvers in FreeGSNKE and Fiesta using EFIT++ reconstructions from MAST-U

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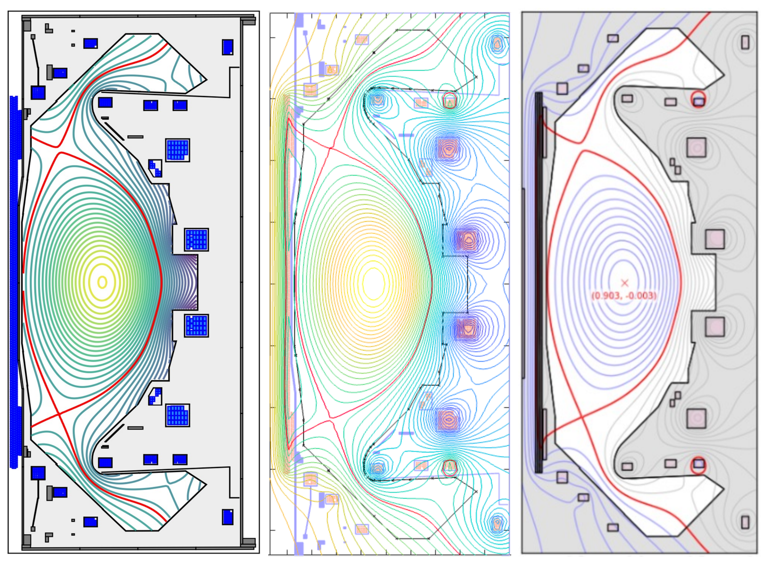
What are we interested in?

Why model plasma equilibria?

- To design, analysis, and control of different plasma scenarios/stability.
- Fast and accurate equilibrium solvers methods are crucial for this.

Our focus:

- Solve the static forward free-boundary Grad-Shafranov (GS) problem.
- Robust validation of static GS solvers, against both analytical solutions and real-world tokamak plasmas.



FreeGSNKE Fiesta EFIT++

What are our aims?

- Validate the static GS solvers in FreeGSNKE and Fiesta by:
  - reproducing equilibria from magnetics-only EFIT++ reconstructions on MAST-U.
  - comparing poloidal fluxes, shape targets, and magnetic readings.

What is the problem?

The static forward (free-boundary) Grad-Shafranov problem:

- A nonlinear elliptic PDE with integral Dirichlet boundary condition.
- Solved for the poloidal flux in a rectangular domain.

Linear elliptic operator: $\Delta^* := R\partial_R R^{-1}\partial_R + \partial_{ZZ}$

Poloidal flux: $\psi(R, Z)$

Toroidal current density: $J_\phi(\psi, R, Z) := J_p(\psi, R, Z) + J_c(R, Z)$

PDE $$\Delta^*\psi = -\mu_0 R J_\phi(\psi, R, Z), \quad (R, Z) \in \Omega$$

BC $$\psi\Big|_{\partial\Omega} = \int_{\Omega_p \cup \Omega_c} G(R, Z; R', Z') J_\phi(\psi, R', Z') \, \mathrm{d}R'\mathrm{d}Z'$$

Plasma domain External conductors Green's function for Δ^*

How is it solved?

- FreeGSNKE uses a Jacobian-free NK method (faster, more stable).
- Fiesta and EFIT++ use Picard iterations (slower, less stable).

What ingredients do we need?

Need consistent inputs (for MAST-U) across all three codes:

1) Accurate machine description:
   a) 12 active poloidal field coils (for plasma shape control).
   b) 150 passive structures (vessel, supports, coil cases, etc.).
   c) Limiter/wall boundary (confines plasma boundary).
2) Currents (in the active coils and passive structures):

$$J_c(R, Z) = \sum_{j=1}^{N_c} \frac{I_j^c(R, Z)}{A_j^c}$$

← Current in conductor j
← Area of conductor j

3) Plasma current density profiles:

$$J_p(\psi, R, Z) = R\frac{\mathrm{d}p}{\mathrm{d}\psi} + \frac{1}{\mu_0 R}F\frac{\mathrm{d}F}{\mathrm{d}\psi}$$

"Lao" profiles

$$\frac{\mathrm{d}p}{\mathrm{d}\tilde{\psi}} = \sum_{i=0}^{n_p} \alpha_i \tilde{\psi}^i - \bar{\alpha}\tilde{\psi}^{n_p+1}\sum_{i=0}^{n_p}\alpha_i$$

$$F\frac{\mathrm{d}F}{\mathrm{d}\tilde{\psi}} = \sum_{i=0}^{n_F} \beta_i \tilde{\psi}^i - \bar{\beta}\tilde{\psi}^{n_F+1}\sum_{i=0}^{n_F}\beta_i$$

Other profiles available: tension spline, Topeol, etc.

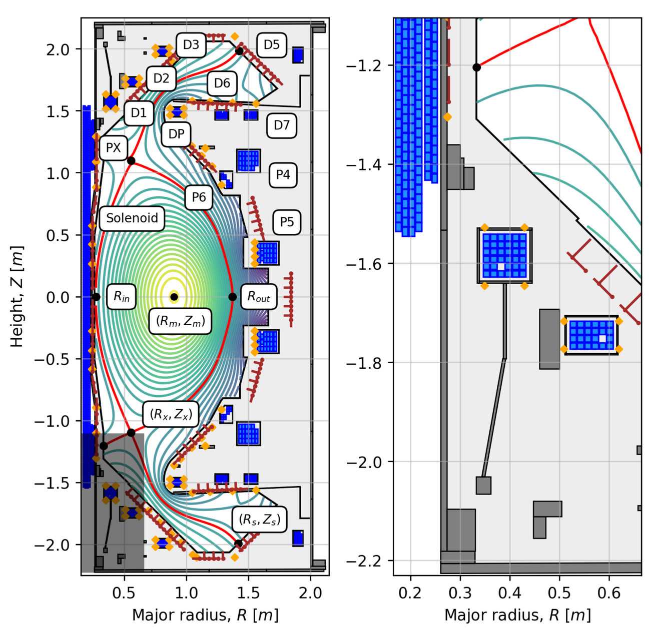


Fig. 1: MAST-U machine in FreeGSNKE with simulated equilibrium of shot 45292 (t = 0.55s).

What were the results?

FreeGSNKE vs. Fiesta vs. EFIT++ (MAST-U shot 45425):

- Conventional divertor, double null, high I_p (~750 kA), ~2.5 MW NBI.
- EFIT++ equilibria used as the "ground truth".
- Excellent match between all quantities of interest and fast simulation!

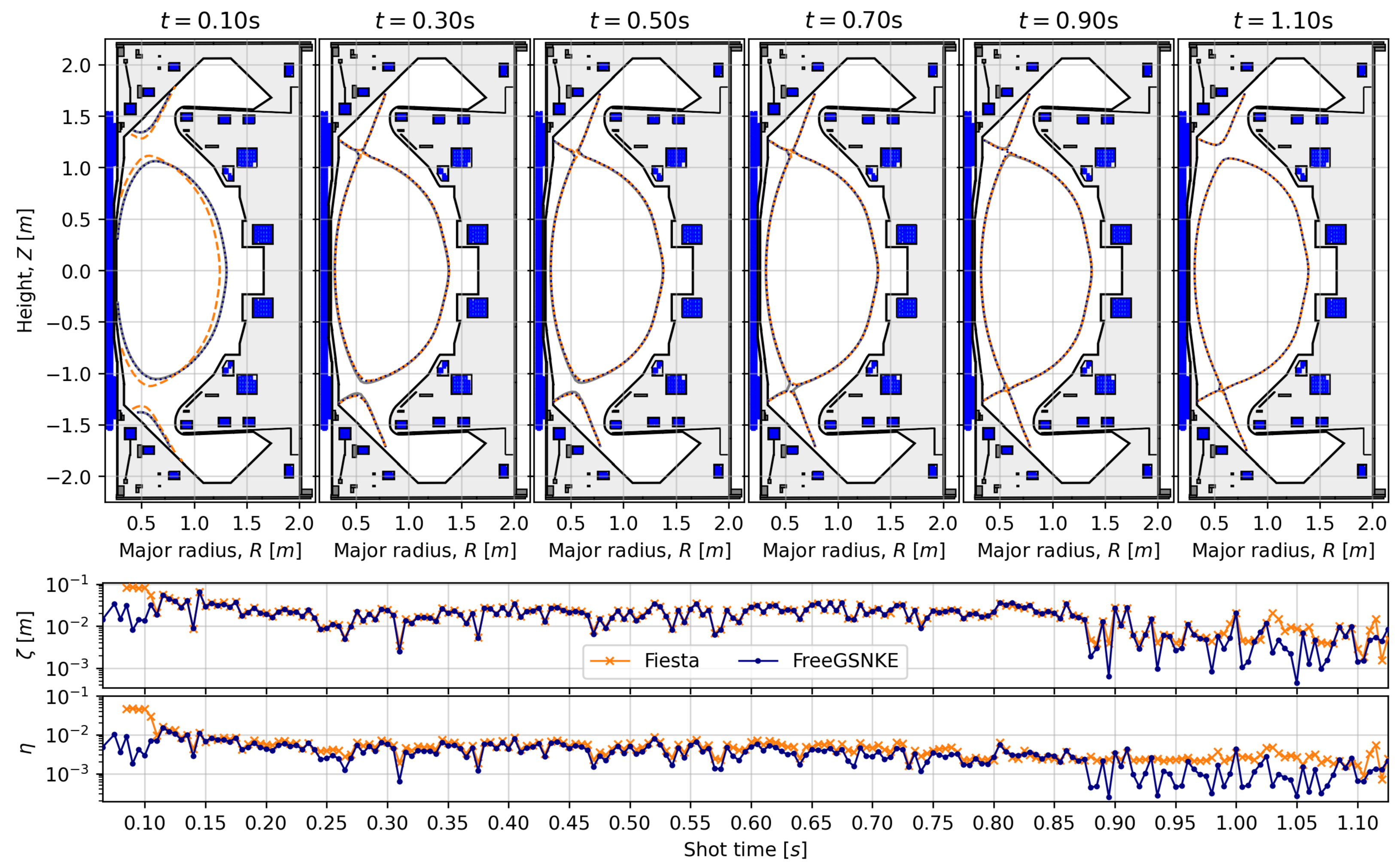


Fig. 2: [Top]: evolution of EFIT++ (solid grey), Fiesta (dashed orange), and FreeGSNKE (dotted blue) separatrices over time. **[Middle]**: max. distance between plasma cores over time (vs. EFIT++). **[Bottom]**: proportion of non-overlapping plasma core areas areas over time (vs. EFIT++).

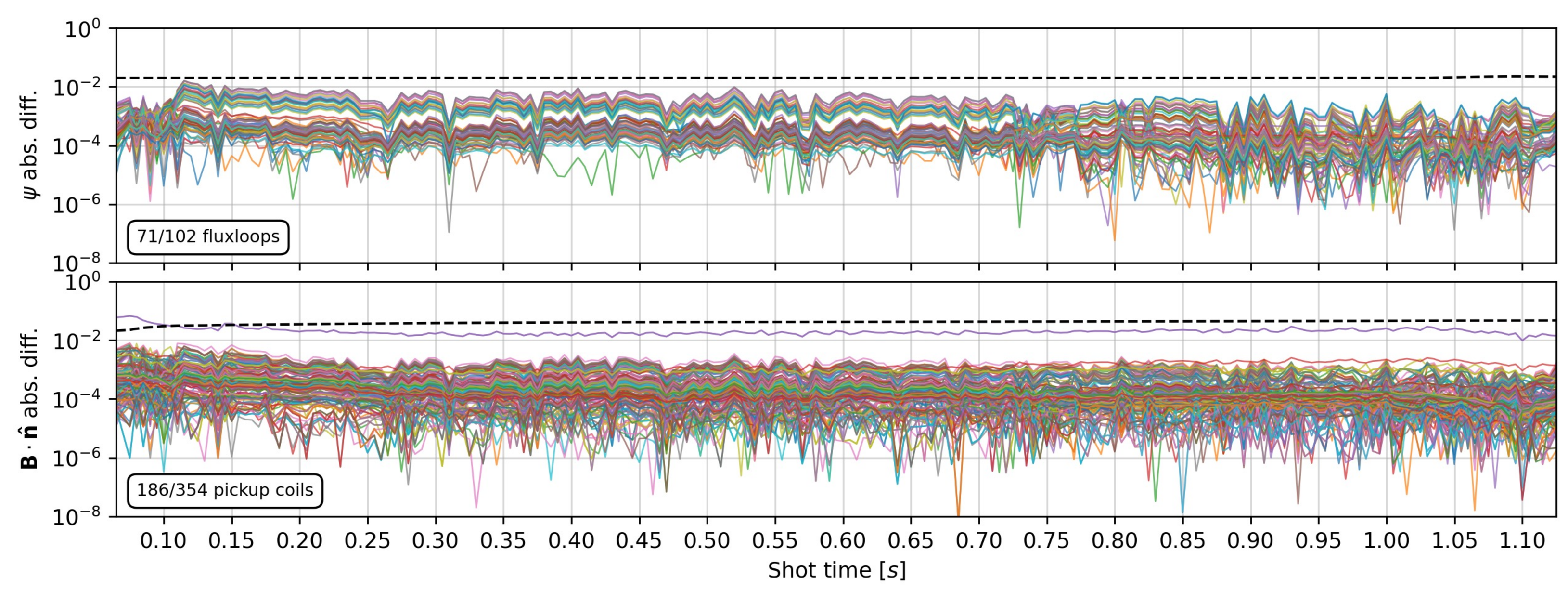


Fig. 3: [Top]: absolute difference between FreeGSNKE and EFIT++ fluxloop measurements. **[Bottom]**: same absolute differences but for the pickup coils (both excluded "faulty" diagnostics).

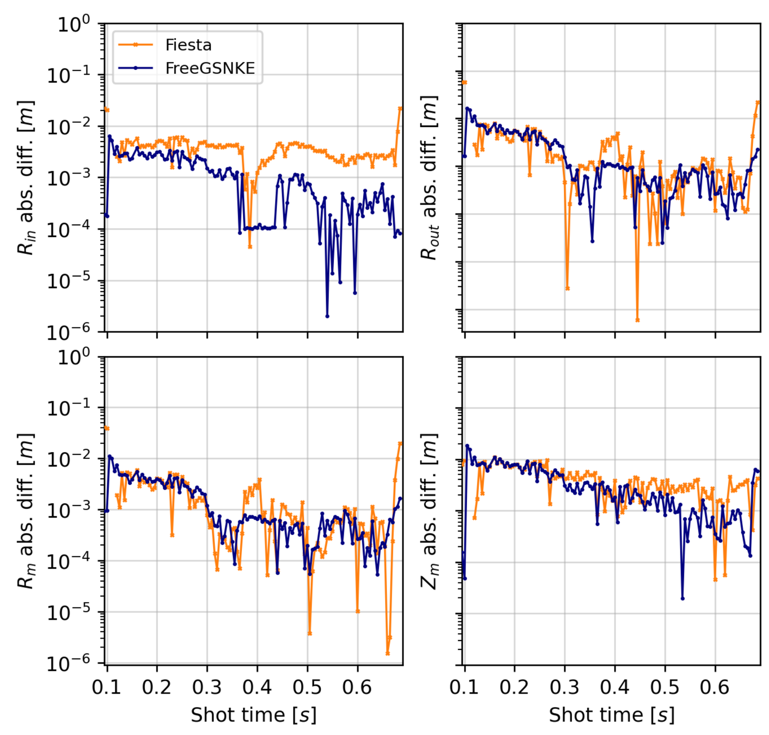


Fig. 4: Absolute differences between EFIT++ and Fiesta (orange)\FreeGSNKE (blue) shape targets. **[Top]**: midplane inner and outer radii. **[Bottom]**: radial and vertical magnetic axis position.

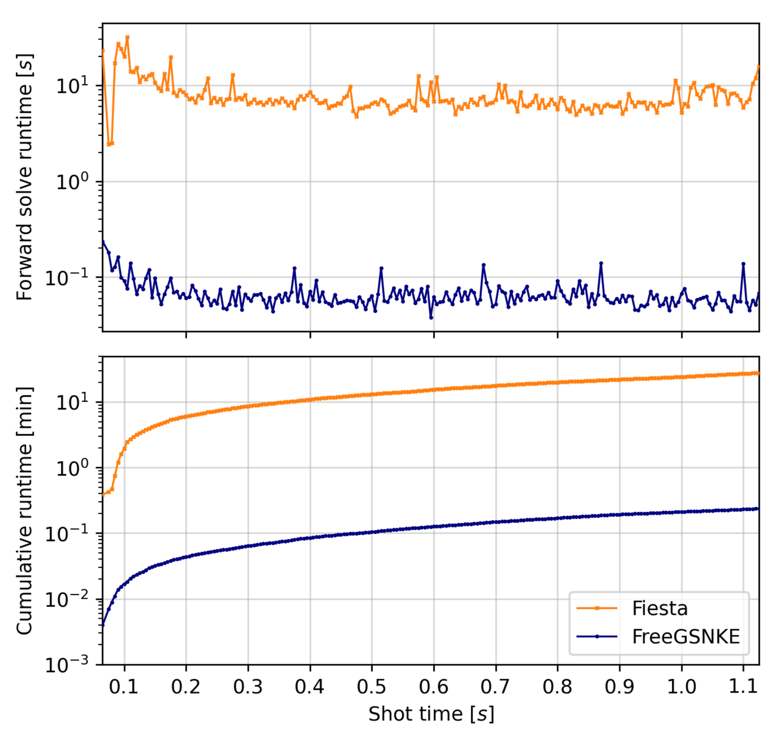


Fig. 5: Fiesta (orange) and FreeGSNKE (blue) runtimes over shot. **[Top]**: runtime per time slice. **[Bottom]**: cumulative runtime (Fiesta: 27 min 48 s, FreeGSNKE: 16 s).

What else are we up to with FreeGSNKE?

Original paper	Static validation	Emulation and control
Amorisco et al. (2024)	Pentland et al. (2024)	Agnello et al. (2024)

- Dynamic solver validation *(in progress)*
- Virtual circuit emulation *(in progress)*
- Porting core solvers to JAX *(in progress)*
- Coupling to transport module *(in progress)*

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